Name————————————, ID ————————

HW3 , Math 531, Spring 2014

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QUESTION 1. (i) Let R be a ring with more than one element such that for each $a \in R$ there is a unique $b \in R$ such that $aba = a$. Prove:

a. R has no zero divisors.

b. If $c \in R$ and $d \in R$ such that $cdc = c$, then $dcd = d$

c. R has an identity

d. R is a division ring

(ii) Let R be an ideal of $\mathbb{Z}$. Show that $I = k\mathbb{Z}$ for some integer k.

(iii) **Freshman Dream** Let R be a commutative ring such that $char(R) = p$ for some prime p. Let $n \geq 1$ be a positive integer and $a, b \in R$. Prove that $(a+b)^{p^n} = a^{p^n} + b^{p^n}$. (Hint: use math induction and note that $(d^m)^v = d^{mv}$)

(iv) Let I be a subset of $R = Z[X]$ such that the constant term of each element of I "lives" in $5Z$. Prove that I is a prime ideal of R.

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